Polytropic and Chaplygin $f(R)$-gravity models

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October 14, 2010

Abstract

Motivated by the recent works of us [1, 2], we establish a correspondence between the $f(R)$-gravity with the polytropic and Chaplygin gas dark energy models. We reconstruct the different $f(R)$-gravity models according to the polytropic, standard Chaplygin, generalized Chaplygin, modified Chaplygin and modified variable Chaplygin gas dark energy models. We obtain the necessary conditions for crossing the phantom divide line of the selected models which describe accelerated expansion of the universe.

Keywords: Modified theories of gravity, Dark energy, Polytropic gas, Chaplygin gas

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1 Friedmann equations in the $f(R)$-gravity

The action of general $f(R)$-gravity is [3]

$$S=\int\sqrt{-g}\mathrm{d}^4x\Big\{\frac{R+f(R)}{2k^2}+L_{\rm matter}\Big\}, \tag{1}$$

where $k^2=8\pi G$. Also G, g, R and $L_{\rm matter}$ are the gravitational constant, the determinant of the metric $g_{\mu\nu}$, the Ricci scalar and the lagrangian density of the matter inside the universe, respectively.

Now if we consider the spatially-flat FRW metric for the universe as

$$\mathrm{d}s^2=-\mathrm{d}t^2+a^2(t)\sum_{i=1}^{3}(\mathrm{d}x^i)^2, \tag{2}$$

and taking $T^{\mu}_{\nu}=\mathrm{diag}(-\rho,p,p,p)$ for the energy-momentum tensor of the matter, then the Friedmann equations in the $f(R)$ theory can be written as [4]

$$\frac{3}{k^2}H^2=\rho_m+\rho_R, \tag{3}$$

$$\frac{1}{k^2}(2\dot{H}+3H^2)=-(p_m+p_R), \tag{4}$$

where

$$\rho_R=\frac{1}{k^2}\left[-\frac{1}{2}f(R)+3(\dot{H}+H^2)f'(R)-18(4H^2\dot{H}+H\ddot{H})f''(R)\right], \tag{5}$$

$$\begin{aligned} p_R=\tfrac{1}{k^2}\Big[&\tfrac{1}{2}f(R)-(\dot{H}+3H^2)f'(R)+\\ &6(8H^2\dot{H}+6H\ddot{H}+4\dot{H}^2+\dddot{H})f''(R)+36(H\ddot{H}+4H^2\dot{H})^2f'''(R)\Big], \end{aligned} \tag{6}$$

with

$$R=6(\dot{H}+2H^2). \tag{7}$$

Here ρ_m and p_m are the energy density and pressure of the matter, respectively. Also ρ_R and p_R are due to the contribution of $f(R)$-gravity.

The energy conservation laws are still given by

$$\dot{\rho}_m+3H(\rho_m+p_m)=0, \tag{8}$$

$$\dot{\rho}_R+3H(\rho_R+p_R)=0. \tag{9}$$

The equation of state (EoS) parameter of the curvature contribution is defined as [5]

$$\begin{aligned} \omega_R&=\frac{p_R}{\rho_R}\\ &=-1+\frac{4\Big(\dot{H}f'(R)+3(3H\ddot{H}-4H^2\dot{H}+4\dot{H}^2+\dddot{H})f''(R)+18(\ddot{H}+4H\dot{H})^2f'''(R)\Big)}{\Big(-f(R)+6(\dot{H}+H^2)f'(R)-36(4H^2\dot{H}+H\ddot{H})f''(R)\Big)}. \end{aligned} \tag{10}$$

For a given $a=a(t)$, by the help of Eqs. (5) and (6) one can reconstruct the $f(R)$-gravity according to any DE model given by the equation of state (EoS) $p=p(\rho)$.

Here we assume a pole-like phantom scale factor as [6]

$$a(t) = a_0(t_s - t)^{-h}, \quad t \leq t_s, \quad h > 0. \tag{11}$$

Using Eqs. (7) and (11) one can obtain

$$\begin{aligned} H^2 &= (\tfrac{\dot{a}}{a})^2 = \tfrac{h}{6(2h+1)}R, \\ a &= a_0\left[\tfrac{R}{6h(2h+1)}\right]^{\frac{h}{2}}. \end{aligned} \tag{12}$$

Substituting Eq. (12) into (5) and (6) gives

$$\rho_R = \frac{1}{2k^2(2h+1)}\Big[-(2h+1)f(R) + (h+1)Rf'(R) - 2R^2f''(R)\Big], \tag{13}$$

$$p_R = \frac{1}{3k^2h(2h+1)}\Big[3h(2h+1)f(R) - h(3h+1)Rf'(R) + 2(2h+3)R^2f''(R) + 4f'''(R)\Big]. \tag{14}$$

Also the EoS parameter of the curvature (10) yields

$$\omega_R = -1 - \frac{2R\Big(hf'(R) - (h-3)Rf''(R) + 2R^2f'''(R)\Big)}{3h\Big((2h+1)f(R) - (h+1)Rf'(R) + 2R^2f''(R)\Big)}. \tag{15}$$

As we mentioned before to solve the above differential equations we have to know the EoS $p = p(\rho)$ or $\rho = \rho(a)$. In the next sections, we reconstruct different $f(R)$-gravities according to the polytropic and different versions of Chaplygin gas dark energy (DE) models.

2 Polytropic $f(R)$-gravity model

Here we reconstruct the $f(R)$-gravity from the polytropic gas DE model. Following [7], the EoS of the polytropic gas is given by

$$p_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}}, \tag{16}$$

where K is a positive constant and n is the polytropic index. Using Eq. (9) the energy density evolves as

$$\rho_\Lambda = \Big(Ba^{\frac{3}{n}} - K\Big)^{-n}, \tag{17}$$

where B is a positive integration constant [7]. Inserting the second relation of Eq. (12) into (17) one can get

$$\rho_\Lambda = \Big(Ba_0^{\frac{3}{n}}(6h(2h+1))^{\frac{-3h}{2n}}\ R^{\frac{3h}{2n}} - K\Big)^{-n}, \tag{18}$$

Equating (13) with (18) gives

$$2R^2f''(R) - (h+1)Rf'(R) - (2h+1)f(R) + \Big(\varepsilon + \xi R^{\frac{3h}{2n}}\Big)^{-n} = 0, \tag{19}$$

where

$$\varepsilon = -K\Big(2(2h+1)k^2\Big)^{\frac{-1}{n}},$$
$$\xi = Ba_0{}^{\frac{3}{n}}\Big(6h(2h+1)\Big)^{\frac{-3h}{2n}}\Big(2(2h+1)k^2\Big)^{\frac{-1}{n}}. \tag{20}$$

Solving Eq. (19) yields the polytropic $f(R)$-gravity as

$$f_{\mathrm{P}}(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{2}{\varepsilon^n(2h+1)\sqrt{1-10h+h^2}} \times$$
$$\left[(m_+)\ {}_2F_1\left(-\frac{2nm_-}{3h}, n; 1-\frac{2nm_-}{3h}; \frac{-\xi R^{\frac{3h}{2n}}}{\varepsilon}\right) - (m_-)\ {}_2F_1\left(-\frac{2nm_+}{3h}, n; 1-\frac{2nm_+}{3h}; \frac{-\xi R^{\frac{3h}{2n}}}{\varepsilon}\right)\right]. \tag{21}$$

where ${}_2F_1$ denotes the first hypergeometric function and

$$m_{\pm} = \frac{3+h \pm \sqrt{h^2-10h+1}}{4}. \tag{22}$$

One notes that to have a real $f(R)$, the parameter h must be in the ranges of $0 < h < 0.1$ and $h > 9.9$.
Replacing Eq. (21) into (15) one can get the following EoS parameter

$$\omega_R = -1 + \frac{1}{1 + \frac{\varepsilon}{\xi} R^{\frac{-3h}{2n}}}\ , \quad h > 0. \tag{23}$$

Using Eqs. (12) and (20), the above relation can be rewritten as

$$\omega_R = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{H}{h}\right)^h\right]^{\frac{-3}{n}} - 1}\ , \quad h > 0. \tag{24}$$

We see that for $\frac{K}{B}\left[a_0\left(\frac{H}{h}\right)^h\right]^{\frac{-3}{n}} > 1$, $\omega_R < -1$ which corresponds to a phantom accelerating universe. The result of Eq. (24) is same as that obtained for the EoS parameter of the polytropic $f(T)$-gravity model given by [2].

3 Standard Chaplygin $f(R)$-gravity model

The EoS of the standard Chaplygin gas (SCG) DE is given by [8]

$$p_\Lambda = -\frac{A}{\rho_\Lambda}, \tag{25}$$

where A is a positive constant. Inserting the above EoS into the energy conservation equation, leads to a density evolving as [8]

$$\rho_\Lambda = \sqrt{A + \frac{B}{a^6}}, \tag{26}$$

where B is an integration constant.

Inserting Eq. (12) into (26) gives

$$\rho_\Lambda = \sqrt{A + ER^{-3h}}, \tag{27}$$

where

$$E = Ba_0{}^{-6}\left(6h(2h+1)\right)^{3h}. \tag{28}$$

Equating (27) with (13) one can obtain

$$2R^2 f''(R) - (h+1)Rf'(R) - (2h+1)f(R) + \sqrt{\varepsilon + \frac{\xi}{R^{3h}}} = 0, \tag{29}$$

where

$$\begin{aligned} \varepsilon &= A\Big(2(2h+1)k^2\Big)^2, \\ \xi &= Ba_0{}^{-6}\left(6h(2h+1)\right)^{3h}\Big(2(2h+1)k^2\Big)^2. \end{aligned} \tag{30}$$

Solving the differential equation (29) yields

$$f_{\rm SCG}(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{2\sqrt{\varepsilon}}{(2h+1)\sqrt{1-10h+h^2}} \times$$
$$\left[(m_+)\,{}_2F_1\left(\frac{m_-}{3h}, \frac{-1}{2}; 1+\frac{m_-}{3h}; \frac{-\xi}{\varepsilon R^{3h}}\right) - (m_-)\,{}_2F_1\left(\frac{m_+}{3h}, \frac{-1}{2}; 1+\frac{m_+}{3h}; \frac{-\xi}{\varepsilon R^{3h}}\right)\right]. \tag{31}$$

Replacing Eq. (31) into (15) one can get the EoS parameter of the standard Chaplygin $f(R)$-gravity model as

$$\omega_R = -1 + \frac{1}{1 + \frac{\varepsilon}{\xi}R^{3h}}. \tag{32}$$

Using Eq. (30) it can be rewritten as

$$\omega_R = -1 + \frac{B\Big(6h(2h+1)\Big)^{3h}}{a_0{}^6 A\ R^{3h} + B\Big(6h(2h+1)\Big)^{3h}}, \tag{33}$$

which for $B < 0$ and $R > 6h(2h+1)a_0{}^{\frac{-2}{h}}\left(\frac{|B|}{A}\right)^{\frac{1}{3h}}$ then ω_R can cross the phantom-divide line.

4 Generalized Chaplygin $f(R)$-gravity model

The EoS of the generalized Chaplygin gas (GCG) DE model is given by [9]

$$p_\Lambda = -\frac{A}{\rho_\Lambda^\alpha}, \tag{34}$$

where α is a constant in the range $0 \leq \alpha \leq 1$ (the SCG corresponds to the case $\alpha = 1$) and A a positive constant. Using Eq. (9), the GCG energy density evolves as [9]

$$\rho_\Lambda = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}, \tag{35}$$

where B is an integration constant. Inserting Eq. (12) into (35) one can get

$$\rho_\Lambda = \left[A + E\ R^{\frac{-3h(1+\alpha)}{2}}\right]^{\frac{1}{1+\alpha}}, \tag{36}$$

where

$$E = B{a_0}^{-3(1+\alpha)}\Big(6h(2h+1)\Big)^{\frac{3h(1+\alpha)}{2}}. \tag{37}$$

Equating (36) with (13), one can obtain

$$2R^2 f''(R) - (h+1)Rf'(R) - (2h+1)f(R) + \left[\varepsilon + \frac{\xi}{R^{\frac{3h\gamma}{2}}}\right]^{\frac{1}{\gamma}} = 0, \tag{38}$$

where

$$\begin{aligned} &\varepsilon = \Big(2(2h+1)k^2\Big)^\gamma A, \\ &\xi = \Big(2(2h+1)k^2\Big)^\gamma B{a_0}^{-3\gamma}(6h(2h+1))^{\frac{3h\gamma}{2}}, \\ &\gamma = 1+\alpha. \end{aligned} \tag{39}$$

Solving the differential equation (38) yields

$$f_{\rm GCG}(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{2\varepsilon^{\frac{1}{\gamma}}}{(2h+1)\sqrt{1-10h+h^2}} \times \\ \left[(m_+)_2F_1\left(\frac{2m_-}{3h\gamma}, \frac{-1}{\gamma}; 1+\frac{2m_-}{3h\gamma}; \frac{-\xi}{\varepsilon R^{\frac{3h\gamma}{2}}}\right) - (m_-)_2F_1\left(\frac{2m_+}{3h\gamma}, \frac{-1}{\gamma}; 1+\frac{2m_+}{3h\gamma}; \frac{-\xi}{\varepsilon R^{\frac{3h\gamma}{2}}}\right)\right]. \tag{40}$$

Replacing Eq. (40) into (15) one can get

$$\omega_R = -1 + \frac{1}{1+\frac{\varepsilon}{\xi}R^{\frac{3h\gamma}{2}}}. \tag{41}$$

Using (39) one can rewrite the above equation as

$$\omega_R = -1 + \frac{B\Big(6h(2h+1)\Big)^{\frac{3h(1+\alpha)}{2}}}{{a_0}^{3(1+\alpha)}A\ R^{\frac{3h(1+\alpha)}{2}} + B\Big(6h(2h+1)\Big)^{\frac{3h(1+\alpha)}{2}}}, \tag{42}$$

which clears that for $B < 0$ and $R > 6h(2h+1){a_0}^{\frac{-2}{h}}\left(\frac{|B|}{A}\right)^{\frac{2}{3h(1+\alpha)}}$ then ω_R can cross the phantom-divide line.

5 Modified Chaplygin $f(R)$-gravity model

The EoS of the modified Chaplygin gas (MCG) DE model is given by [10]

$$p_\Lambda = A\rho_\Lambda - \frac{B}{\rho_\Lambda^\alpha}, \tag{43}$$

where A and B are positive constants and $0 \leq \alpha \leq 1$. Using Eq. (9), the MCG energy density evolves as [10]

$$\rho_\Lambda = \left(\frac{B}{1+A} + \frac{C}{a^{3(1+\alpha)(1+A)}}\right)^{\frac{1}{1+\alpha}}, \tag{44}$$

where C is an integration constant. Inserting Eq. (12) into (44) one can get

$$\rho_\Lambda = \left[\frac{B}{1+A} + E\ R^{\frac{-3h(1+\alpha)(1+A)}{2}}\right]^{\frac{1}{1+\alpha}}, \tag{45}$$

where

$$E = C{a_0}^{-3(1+\alpha)(1+A)}\Big(6h(2h+1)\Big)^{\frac{3h(1+\alpha)(1+A)}{2}}. \tag{46}$$

Equating (45) with (13) yields

$$2R^2 f''(R) - (h+1)Rf'(R) - (2h+1)f(R) + \Big[\varepsilon + \frac{\xi}{R^{\frac{3h\gamma\eta}{2}}}\Big]^{\frac{1}{\gamma}} = 0, \tag{47}$$

where

$$\begin{aligned}
\varepsilon &= B(1+A)^{-1}\Big(2(2h+1)k^2\Big)^{\gamma},\\
\xi &= C a_0^{-3\gamma\eta}(6h(2h+1))^{\frac{3h\gamma\eta}{2}}\Big(2(2h+1)k^2\Big)^{\gamma},\\
\gamma &= 1+\alpha,\\
\eta &= 1+A.
\end{aligned} \tag{48}$$

Solving Eq. (47) gives

$$f_{\rm MCG}(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{2\varepsilon^{\frac{1}{\gamma}}}{(2h+1)\sqrt{1-10h+h^2}} \times$$
$$\left[(m_+)_2F_1\left(\frac{2m_-}{3h\gamma\eta}, \frac{-1}{\gamma}; 1+\frac{2m_-}{3h\gamma\eta}; \frac{-\varphi}{\varepsilon R^{\frac{3h\gamma\eta}{2}}}\right) - (m_-)_2F_1\left(\frac{2m_+}{3h\gamma\eta}, \frac{-1}{\gamma}; 1+\frac{2m_+}{3h\gamma\eta}; \frac{-\xi}{\varepsilon R^{\frac{3h\gamma\eta}{2}}}\right)\right]. \tag{49}$$

Replacing Eq. (49) into (15) gets

$$\omega_R = -1 + \frac{1+A}{1+\frac{\varepsilon}{\xi}R^{\frac{3h\gamma\eta}{2}}}. \tag{50}$$

Using Eq. (48) it reduces to

$$\omega_R = -1 + \frac{C(1+A)^2\Big(6h(2h+1)\Big)^{\frac{3h(1+\alpha)(1+A)}{2}}}{{a_0}^{3(1+\alpha)(1+A)}B\ R^{\frac{3h(1+\alpha)(1+A)}{2}} + C(1+A)\Big(6h(2h+1)\Big)^{\frac{3h(1+\alpha)(1+A)}{2}}}, \tag{51}$$

which shows that for $C < 0$ and $R > 6h(2h+1){a_0}^{\frac{-2}{h}}\left[\frac{|C|(1+A)}{B}\right]^{\frac{2}{3h(1+\alpha)(1+A)}}$ then ω_R can cross the phantom-divide line.

6 Modified Variable Chaplygin $f(R)$-gravity model

The EoS of the modified variable Chaplygin gas (MVCG) DE model is given by [11]

$$p_\Lambda = A\rho_\Lambda - \frac{B}{a^n \rho_\Lambda^\alpha}, \tag{52}$$

where A, B and n are positive constants and $0 \le \alpha \le 1$. Using Eq. (9), the MCG energy density evolves as [10]

$$\rho_\Lambda = \frac{1}{a^{\frac{n}{1+\alpha}}} \left(\frac{3(1+\alpha)B}{3(1+\alpha)(1+A)-n} + \frac{C}{a^{3(1+\alpha)(1+A)-n}} \right)^{\frac{1}{1+\alpha}}, \tag{53}$$

where C is an integration constant. Inserting Eq. (12) into (53) one can get

$$\rho_\Lambda = L\ R^{\frac{-nh}{2(1+\alpha)}} \left[M + N\ R^{\frac{h}{2}\left(n-3(1+\alpha)(1+A)\right)} \right]^{\frac{1}{1+\alpha}}, \tag{54}$$

where

$$\begin{aligned} L &= a_0^{-\frac{n}{1+\alpha}} \Big[6h(2h+1)\Big]^{\frac{nh}{2(1+\alpha)}}, \\ M &= \tfrac{3(1+\alpha)B}{3(1+\alpha)(1+A)-n}, \\ N &= C a_0^{\left(n-3(1+\alpha)(1+A)\right)} \Big[6h(2h+1)\Big]^{\frac{h}{2}\left(3(1+\alpha)(1+A)-n\right)}. \end{aligned} \tag{55}$$

Equating (54) with (13) gives

$$2R^2 f''(R) - (h+1)Rf'(R) - (2h+1)f(R) + \frac{1}{R^\sigma}\Big[\varepsilon + \frac{\xi}{R^\delta}\Big]^{\frac{1}{\gamma}} = 0, \tag{56}$$

where

$$\begin{aligned} \varepsilon &= \tfrac{3(1+\alpha)B\big[6h(2h+1)\big]^{\frac{nh}{2}}\big[2(2h+1)k^2\big]^\gamma}{a_0^n\big[3(1+\alpha)(1+A)-n\big]}, \\ \xi &= \tfrac{C\big[6h(2h+1)\big]^{\frac{3h}{2}(1+\alpha)(1+A)}\big[2(2h+1)k^2\big]^\gamma}{a_0^{3(1+\alpha)(1+A)}}, \\ \gamma &= 1+\alpha, \\ \delta &= \tfrac{3h(1+\alpha)(1+A)}{2} - \tfrac{nh}{2}, \\ \sigma &= \tfrac{nh}{2(1+\alpha)}. \end{aligned} \tag{57}$$

Solving the differential equation (56) yields

$$\begin{aligned} f_{\rm MVCG}(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{2\varepsilon^{\frac{1}{\gamma}}}{(h(2+\sigma)+(1+\sigma)(1+2\sigma))\sqrt{1-10h+h^2}} \times \\ \Big[(m_+ + \sigma)_2F_1\Big(\frac{m_- + \sigma}{\delta}, \frac{-1}{\gamma}; 1 + \frac{m_- + \sigma}{\delta}; \frac{-\xi}{\varepsilon R^\delta}\Big) \\ -(m_- + \sigma)_2F_1\Big(\frac{m_+ + \sigma}{\delta}, \frac{-1}{\gamma}; 1 + \frac{m_+ + \sigma}{\delta}; \frac{-\xi}{\varepsilon R^\delta}\Big)\Big]. \end{aligned} \tag{58}$$

Replacing Eq. (58) into (15) yields the EoS parameter as

$$\omega_R = -1 + \frac{2}{3h}\left[\sigma + \frac{\delta\xi}{\gamma(\xi + \varepsilon R^\delta)}\right]. \tag{59}$$

With the help of Eq. (57) it yields

$$\omega_R = -1 + \frac{1}{3(1+\alpha)}\left[n + \frac{4\delta^2 C\left(a_0^{\frac{-2}{h}}6h(2h+1)\right)^\delta}{3h^2(1+\alpha)B\ R^\delta + 2h\delta C\left(a_0^{\frac{-2}{h}}6h(2h+1)\right)^\delta}\right], \tag{60}$$

which shows that for $C < 0$ and $R > 6h(2h+1){a_0}^{\frac{-2}{h}}\left[\frac{4|C|\delta(1+A)(2nh-\delta)}{3nh(1+\alpha)B}\right]^{\frac{1}{\delta}}$ then the EoS parameter of the $f_{\rm MVCG}(R)$-gravity model corresponds to a phantom accelerating universe.

7 Conclusions

Here we considered the polytropic gas, SCG, GCG, MCG and MVCG models of the DE. We reconstructed the different theories of modified gravity based on the $f(R)$ action in the spatially-flat FRW universe and according to the selected DE models. We also obtained the EoS parameter of the polytropic gas, standard Chaplygin, generalized Chaplygin, modified Chaplygin and modified varaibel $f(R)$-gravity scenarios. We showed that crossing the phantom-divide line can occur when the constant parameters of the models to be chosen properly.

Acknowledgements
The work of K. Karami has been supported financially by Research Institute for Astronomy & Astrophysics of Maragha (RIAAM), Maragha, Iran.

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